Compression Stiffness of Bonded Rubber Layers — Shape Factor Method

1. Theory

The compressive stiffness of a bonded rubber layer depends strongly on the shape factor (S), defined as the ratio of loaded area to total free bulge area. For a circular layer with outer radius r_o, inner radius r_i, and thickness t:

$$S = (r_o - r_i) / (2t)$$

For small strains and nearly incompressible rubber ($\nu \approx 0.5$), Gent and Lindley derived simplified relations for the effective compressive modulus E_c:

Bonded loaded surfaces (no slip): $E_c = 6 \cdot G \cdot S^2$
Unbonded loaded surfaces (lubricated): $E_c = 4 \cdot G \cdot S^2$

And a simplified version for bonded loaded surfaces: $E_c = G(1+2S^2)$

where G is the shear modulus. The total axial compression stiffness is then:

$$K_c = E_c \cdot A / t = Ec \cdot \pi \cdot (r_o^2 - r_i^2) / t$$

Finite compressibility can be included with a mild reduction factor, e.g., $(K / (K + \alpha \cdot G))$ with $\alpha \approx 1$–2. When $K \gg G$ (typical of rubber), this factor is ~1 and the incompressible formulas above are adequate.

2. References

1) Gent, A. N., & Lindley, P. B. (1959). The compression of bonded rubber blocks. Proceedings of the Institution of Mechanical Engineers, 173(1), 111–122.
2) Lindley, P. B. (1970). Load–deflection relationships for bonded rubber blocks. Journal of Strain Analysis, 5(3), 190–195.
3) Gent, A. N. (2012). Engineering with Rubber (3rd ed.). Hanser Publishers.

3. Validity Range and Limitations

These shape-factor relations are empirical small-strain approximations. They are generally reliable for circular pads with a moderate shape factor and small axial strain ($\Delta/t \lesssim 0.1$).

Shape Factor (S)	Typical Behavior	Applicability
S < 0.5	Shear-dominated, bulging largely unconstrained	Not accurate; use shear estimate $Kc \approx G \cdot A/t$

$0.5 \le S \le 5$	Moderate constraint typical of isolators/pads	Accurate for small strains ($\Delta/t < 0.1$)
$S > 5–10$	Highly constrained; approaches hydrostatic limit	Underpredicts; include finite-K or use FE

Note: At very large S, the response tends toward the hydrostatic limit ($E_c \rightarrow 3K$). At very small S, simple shear estimates are more appropriate than S^2 scaling.

4. Assumptions

• Small strain, linear elastic rubber with shear modulus G.
• Incompressible limit unless otherwise noted; for finite compressibility a factor $K/(K+\alpha G)$ ($\alpha \approx 1–2$) may be applied.
• Circular geometry with uniform thickness; end conditions either fully bonded (no slip) or lubricated (free).
• Units: inches and pounds-force; Kc reported in lbf/in.

5. Comparison: Gent–Lindley ($6GS^2$) vs Simplified ($1 + 2S^2$)

Finite element (FE) simulations were compared to two analytical stiffness relations:
1) The Gent–Lindley model ($E_c = 6 \cdot G \cdot S^2$), and
2) A simplified empirical interpolation ($E_c = G \cdot (1 + 2 \cdot S^2)$).

These formulas differ primarily in how they represent the effect of lateral constraint at moderate shape factors.

The Gent–Lindley expression assumes nearly complete suppression of lateral bulging, leading to a quasi-hydrostatic stress state. It performs well for moderate constraint ($0.5 \le S \le 3$) but tends to overpredict stiffness for $S > 3–4$, because real bonded layers experience partial bulging near their edges. For small S (< 1), it underpredicts stiffness since shear deformation dominates rather than volumetric compression.

The simplified ($1 + 2S^2$) relation, attributed to Lindley and other subsequent authors, provides a smooth interpolation between the pure shear limit ($S \rightarrow 0$) and the highly constrained limit ($S \rightarrow \infty$). For small S, $E_c \approx G$, which correctly represents shear deformation. For large S, $E_c \approx 2 \cdot G \cdot S^2$, which aligns closely with FE and experimental data for practical pads ($S \le 6$). This model captures the realistic transition between shear- and compression-dominated behavior.

Model	Expression for Ec	Behavior / Applicability
Gent–Lindley	$E_c = 6 \cdot G \cdot S^2$	Fully bonded; overpredicts for large S; underpredicts for S < 1; good for $0.5 \le S \le 3$

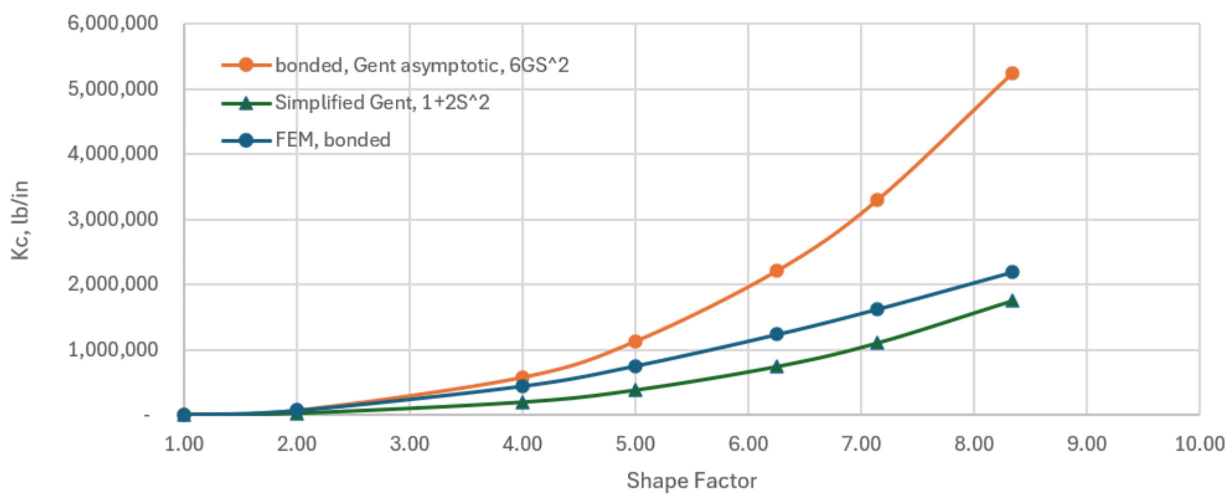
Simplified (Lindley-type)	$E_c = G \cdot (1 + 2 \cdot S^2)$	Interpolates between shear (S→0) and constraint (S→∞); fits FE for 0 ≤ S ≤ 6

The FE stiffness results generally align more closely with the $G \cdot (1 + 2 \cdot S^2)$ formulation than with the Gent–Lindley $6 \cdot G \cdot S^2$ expression. This supports the interpretation that real bonded rubber elements allow partial lateral bulging rather than complete constraint. The Gent–Lindley equation therefore provides an upper bound, while the simplified interpolation better represents average behavior.

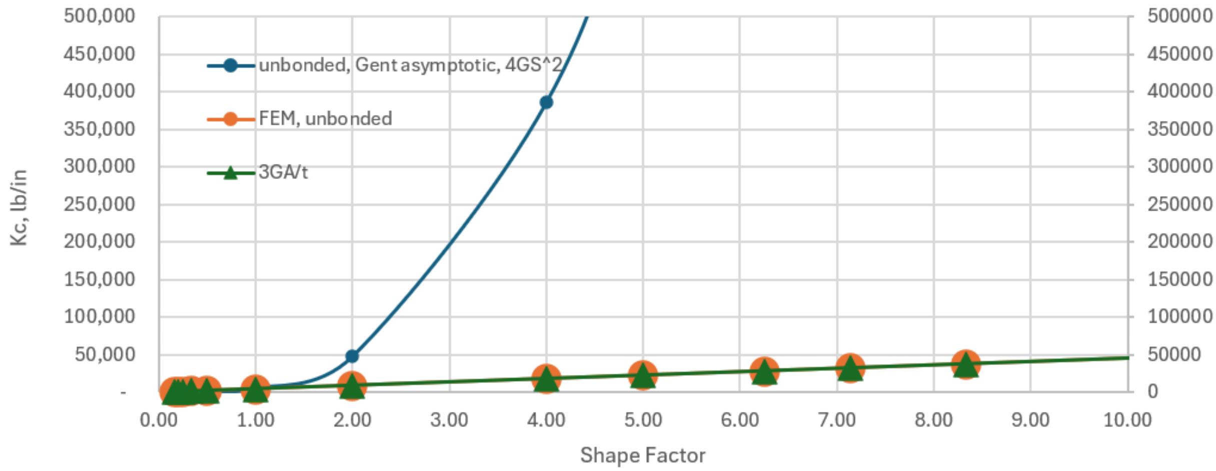
6. FEM and Gent/Lindley Methods Comparison

The chart below shows the comparison of rubber pad stiffness for the two Gent/Lindley methods and the FEM model. The rubber pad for this data was 4 inches OD with no center hole. Bulk modulus was 180,000 psi. Shear modulus 120 psi. The thickness, t, was varied to vary the shape factor, S. The loaded surfaces were constrained in the radial direction (bonded).

As noted above the $6GS^2$ model is an upper bound. The FEM model predicts slightly higher E_c's than the simplified model.



The chart below show results for the "unbonded" condition. The Gent/Lindley method substantially overpredicts compression Ec for this condition for truly free loaded surfaces. The FEM model matches the 3GA/t exactly. In fact, for the ideal case of unbonded surfaces there is no dependence on shape factor. In reality reaching truly free surfaces maybe tough so the 3GA/t can be treated as a lower bound.


unbonded, Gent asymptotic, 4GS^2
FEM, unbonded
3GA/t
Kc, lb/in
Shape Factor
500,000
450,000
400,000
350,000
300,000
250,000
200,000
150,000
100,000
50,000
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500000
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