SDOF Drop Shock Module

1. Overview

The SDOF Drop Shock module models the vertical impact response of a single rigid mass dropped from a specified height onto a linear spring and viscous damper. The module is intended for preliminary shock and impact analysis where the system can be reasonably idealized as a single-degree-of-freedom (SDOF) system.

2. User Inputs

The user specifies the following inputs:

- W – Weight of the mass
- h – Drop height
- k – Linear spring rate
- c – Viscous damping coefficient
- g – Gravitational acceleration
- V_0 – Initial velocity at release (optional)

All derived quantities are updated immediately as inputs are modified.

3. Outputs

The module computes and displays:

- Displacement history of the mass
- Acceleration history (reported in g's)
- Natural frequency (f_n)
- Critical damping coefficient (c_c)
- Damping ratio (ζ)
- Time to initial contact
- Maximum displacement
- Maximum acceleration

The animation and plots automatically stop after 20 natural cycles following first contact.

4. Solution Technique

The system is solved in two phases:

1. Free-fall phase
   The mass undergoes uniform acceleration under gravity until contact with the spring.

2. Contact phase
   Once contact occurs, the governing equation of motion is:

$$m \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x = W$$

where x is displacement measured from the contact point.

The contact-phase response is integrated numerically using a fourth-order Runge–Kutta (RK4) scheme. The time step is selected based on the system natural frequency to ensure numerical stability and accuracy.

5. Validation Case (Undamped System)

A simple validation case is obtained by setting the damping coefficient $c = 0$.

For this case, energy conservation provides a closed-form check on the maximum compression.

At maximum compression, the initial gravitational potential energy of the mass equals the strain energy stored in the spring:

$$W \cdot h = (1/2) \cdot k \cdot x_{max}^2$$

Solving for maximum compression:

$$x_{max} = \sqrt{(2 \cdot W \cdot h / k)}$$

The maximum displacement reported by the numerical solution should closely match this value, providing a direct validation of the implementation.

6. Assumptions and Limitations

- Linear spring behavior
- Linear viscous damping
- Rigid mass (no internal flexibility)
- One-dimensional vertical motion

The model is intended for preliminary analysis and conceptual design rather than detailed high-fidelity impact simulations.

7. References

1. Harris, C. M., and Piersol, A. G., *Harris' Shock and Vibration Handbook*, McGraw-Hill.
2. Den Hartog, J. P., *Mechanical Vibrations*, Dover Publications.
3. Thomson, W. T., and Dahleh, M. D., *Theory of Vibration with Applications*, Prentice Hall.
4. Goldsmith, W., *Impact: The Theory and Physical Behaviour of Colliding Solids*, Dover Publications.